Classroom Worksheet-1 IIT-JEE ADVANCED

The answer to each of the questions are upto 4 digits. (Integer Type)

1. For a positive integer n, let $I_n = \int_{-\pi}^{\pi}\left(\frac{\pi}{2}-|x|\right)\cos nx\, dx$. Find the value of $9(I_1 + I_2 + I_3 + I_4)$.

2. A focal chord *PQ* is drawn to the parabola $y^2 = 4\alpha x$ which makes an angle $\phi = \sec^{-1}2$ with the x-axis. If S is the focus and $PS = (\cot^2\theta)\, SQ$ then find the number of values of θ in interval $(0, 2\pi)$.

3. Consider 4 independent trials in which an event A occurs with probability $\frac{1}{3}$. The event B will occur with probability 1 if the event A occurs at least twice, it cannot occur if the event A does not occur and it occurs with a probability $\frac{1}{2}$ if the event A occurs once. If the probability P of the occurrence of event B can be expressed as $\frac{m}{n}$, find the least value of (m + n) ,where m, n ∈ N.

4. A circle of radius 1 unit touches the positive X-axis and positive Y-axis at P and *Q* respectively. A variable line L passing through the origin intersects the circle in two points M and N. If m is the slope of the line L for which the area of the triangle MNQ is maximum, then find the value of $2010(m^2)$.

5. Let *a*, b, c be real numbers such that a + b + c = 6 and ab + bc + ca = 9. If exactly one root of the equation $x^2 - (m + 2)\, x + 5m = 0$ lies between minimum and maximum value of c, then find the number of integral values of m.

6. Find the number of points of intersection of the curves y = cos x , y = sin 3*x* , if $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$.

7. If the value of the definite integral $\int_0^{\ln 2}(x-\ln 2)e^x \cdot e^{\ln 2} \cdot e^{\ln\left(1+e^x\right)^{-2}}\, dx = \ln\left(\frac{a}{b}\right)$ where $\frac{a}{b}$ is rational in the lowest form, then find (b – *a*).

8. Find the sum of all possible values of α in [1, 100] for which the function $f(x) = 2x^3 - 3(2 + \alpha)\, x^2 + 12\alpha x$ has exactly one local maximum and one local minimum.

9. If $\sum_{n=0}^{\infty} 2\, arc\cot\left(\frac{n^2+n+4}{2}\right) = k\pi$, then find the value of k.

10. If the range of values of a for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of the equation $x^2 - 2(a + 1)\, x + a(a - 1) = 0$ is (p, q) , find the value of $\left(q + \frac{1}{p^2}\right)$.

11. Find the number of ordered pairs (*x*, y) satisfying the equations
sin *x* cos y = 1 & $x^2 + y^2 \le 9\pi^2$

12. Let M be a 2 × 2 matrix such that $M\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ & $M^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$. If x_1 & $x_2(x_1 > x_2)$ are the two values of x for which det(M – xI) = 0, where I is an identity matrix of order 2 then find the value of $(5x_1 + 2x_2)$.

13. Let $f(x)=(p\cos x+q\sin x)(x^2+\alpha x+\beta)$. If $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx$ vanishes for all real values of p and q then find the value ($\pi^2 + 4\beta + \alpha$).

14. If the three vectors $\vec{V}_1=\hat{i}-a\,\hat{j}-a\,\hat{k}$, $\vec{V}_2=b\hat{i}-\hat{j}+b\,\hat{k}$, $\vec{V}_3=c\hat{i}+c\,\hat{j}-\hat{k}$ are linearly dependent then find the value of $(1 + a)^{-1}$+ (1 + b))$^{-1}$ + (1 +c))$^{-1}$. (single digit)

15. Let P and Q are two points in xy-plane on the curve y = x^7 – 2x^5 + 5x^3 + 8x + 5 such that $\overrightarrow{OP}\cdot\hat{i}=2$ & $\overrightarrow{OQ}\cdot\hat{i}=-2$ and the magnitude of $\overrightarrow{OP}+\overrightarrow{OQ}\cdot j=2M$ (where O is origin) then find the value of M.

16. If the number of ordered pairs (S,T) of subsets of {1, 2, 3, 4, 5, 6) are such that S ∪ T contains exactly three elements is 10λ, then find the value of λ. (2 digits)

17. Let $\vec{a}$ & $\vec{b}$ be two vectors such that $|\vec{a}|=\frac{1}{2}=|\vec{b}|$. If λ denotes the minimum value of $\frac{1}{|\vec{a}+\vec{b}|^2}+\frac{1}{|\vec{a}-\vec{b}|^2}$, then find the number of solution of equation sec$^2\phi$ = λ in interval (–4π , 8π].

18. In a football tournament a team T has to play with each of the 6 other teams once. Each match can result in a wins, draw or loss. Find the number of ways in which the term T finishes with more wins than losses.

19. A line with direction cosines proportional to 2, 1, 2 meets the lines x = y + 2 = z and x + 2 =2y = 2z at P and Q respectively. Find the distance between P and Q.

20. Let k be a positive integer and f (x) be a polynomial with integer coefficients satisfying $2\int_1^x f(t)dt + x^k = x\,f(x)$, where $x \geq 1$. Find the sum of all possible values of k.

21. The first term of a geometric progression is equal to b – 2 ,the third term is b + 6 and arithmetic mean of the first and third term to the second term is in the ratio 5 : 3. Find the positive integral value of b.

22. If the values of y satisfying the equation x^2 – 2x sin(xy) + 1 = 0 is expressed in the form of kπ (k ∈ R) then find the sum of all possible values of k in (0, 48).